

Magnetic impurities in a superconductor: Effect of domain walls and interference

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We consider the effect of magnetic impurities, modeled by classical spins, in a conventional superconductor. We study their effect on the quasiparticles, specifically on the spin density and local density of states (LDOS). As previously emphasized, the impurities induce multiple scatterings of the quasiparticle wave functions leading to complex interference phenomena. Also, the impurities induce quantum phase transitions in the many-body system. Previous authors studied the effect of either a small number of impurities (from one to three) or a finite concentration of impurities, typically in a disordered distribution. In this work we assume a regular set of spins distributed inside the superconductor in such a way that the spins are oriented, forming different types of domain walls, assumed stable. This situation may be particularly interesting in the context of spin transfer due to polarized currents traversing the material.

I. INTRODUCTION

The effect of perturbing the superconducting state is a topic of interest since often it provides information about the superconducting state, its nature and origin, but also it provides information about its parent normal state. This has gained particular interest due to the elusive nature of pairing in high-temperature superconductors. In particular, the effect of magnetic fields in its various forms has attracted interest for a long time, such as the effect of vortices in type-II superconductors and in general the interplay between superconductivity and magnetic field.

Also, the effect of impurities has been studied considering both non-magnetic and magnetic impurities in conventional and unconventional superconductors¹. In the case of non-magnetic impurities and s -wave pairing, Anderson's theorem states that, at least for low concentrations, they have little effect since the impurities are not pair-breaking². In d -wave superconductors however, non-magnetic impurities cause a strong pair breaking effect³. In the limit of strong scattering it was found that the lowest energy quasiparticles become localized below the mobility gap, even in a regime where the single-electron wave functions are still extended⁴. This result has been confirmed solving the Bogoliubov-de Gennes equations with a finite concentration of non-magnetic impurities⁵. However, allowing for angular dependent impurity scattering potentials it has been found that the scattering processes close to the gap nodes may give rise to extended gapless regions⁶.

Several conflicting predictions have appeared in the literature regarding the effect of the presence of impurities. Some progress towards understanding the disparity of theoretical results has been achieved realizing that the details of the type of disorder significantly affect the density of states⁷. Particularly in the case of d -wave superconductors, in contrast to conventional gaped s -wave superconductors, the presence of gapless nodes is expected to affect the transport properties. Using a field theoretic description and linearizing the spectrum around the four Dirac-like nodes it has been suggested that the system is critical. It was obtained that the density of states is of the type $\rho(\epsilon) \sim |\epsilon|^\alpha$, where α is a non-universal exponent dependent on the disorder, and that the low energy modes are extended states (critical metal)⁸. Taking into account the effects of inter-nodal scattering (hard-scattering) it has been shown that an insulating state is obtained instead, where the density of states still vanishes at low energy but with an exponent $\alpha = 1$ independent of disorder⁹. Using the Bogoliubov-de Gennes (BdG) equations it was found that the d -wave superconductivity is mainly destroyed locally near a strong scatterer. The superfluid density is strongly suppressed near the impurities but only mildly affected elsewhere¹⁰. No evidence for localization of the low energy states was found. The superfluid density is suppressed but less than expected^{5,11} and, accordingly, the decrease of the critical temperature with disorder is much slower than previously expected, in accordance with experiments¹². Similar results of an inhomogeneous order parameter were also obtained for s -wave superconductors¹³. The results show that the order parameter is only significantly affected close to the impurity locations.

On the other hand, magnetic impurities induce in-gap bound states in conventional superconductors, while for instance in d -wave superconductors, due to their gapless nature, they just induce virtual bound states. The local nature of these boundstates can now be studied in detail due to the progress in experimental techniques like Scanning Tunneling Microscopy (STM). In their pioneer work,¹⁴ Abrikosov and Gor'kov considered the properties of a superconductor with magnetic impurities. They demonstrated that the noninteracting magnetic impurities suppress the superconductivity, so that at some critical impurity density the superconducting gap Δ shrinks to zero, which was later identified as a quantum critical point¹⁵.

It was also shown long ago that the presence of few magnetic impurities (vanishing concentration) is enough to lead to rather interesting quantum phase transitions, in particular in the total magnetic moment of the condensate¹⁶. In the simplest case the magnetic impurities can be treated as classical spins inserted in the superconductor, acting as local magnetic fields. Indeed it was shown that if the coupling is weak enough, the Kondo coupling can be overlooked and a much simpler treatment of the classical case provides a good description¹⁷. Actually, even though there are similarities between the interaction of the electrons with an impurity both in the standard Kondo effect in metals and the coupling to the magnetic impurity in the superconductor, while the Kondo effect is merely a crossover between a free spin at high energies or temperatures and the Kondo singlet (for a $S = 1/2$ impurity in a single s band) at low energies¹⁸, in the case of the superconductor there is a true first order quantum phase transition¹⁹. The phase transition occurs through a level crossing between two states as the coupling between the spin density of the electrons and the impurity spin grows. The level crossing occurs between one state that describes an uncompensated local spin (at smaller coupling) and a state where the impurity spin is compensated (partially since for the classical description to be valid the spin has to be large).

The case of two (or three) impurities has also been studied^{20,21} and quantum phase transitions have been identified both as a function of coupling but also as a function of other parameters such as the distance between the impurities, or the angle between their spin orientations. Varying judiciously these parameters one may cross various phase transition points. The richness of these alternations was explained in terms of interference effects between the locally induced states at the impurity sites, in a related way to the study of mirages and other interference effects in other systems^{21,22,23,24,25}.

In the case of interacting magnetic moments in a superconductor, the nature of the superconducting transition changes, and the critical temperature slightly increases²⁶. Recently, Galitski and Larkin studied²⁷ the effect of a spin-glass ordering of magnetic impurities on the superconductivity. They showed that the superconducting properties depend on the state of the magnetic system and found a shift of the superconducting quantum critical point. A related problem is the mechanism of the exchange interaction of magnetic moments in the superconductor. In the case of normal metal, this is the RKKY interaction due to Friedel oscillations of the magnetic density in the electron gas, induced by a single moment. In the case of a superconductor, the RKKY interaction is affected by the gap at the Fermi surface²⁸. This effect was revisited in some recent works^{27,29}. The main result is that the form of the RKKY interaction in superconductors is mainly preserved but the interaction contains a decaying exponential factor which vanishes when $\Delta \rightarrow 0$.

As is well known, in the case of dominating ferromagnetic long-range ordering of the moments, the competition between superconductivity and ferromagnetism leads to the absence of superconductivity. In other words, ferromagnetism suppresses the superconductivity acting like an external magnetic field. However, it is possible to reach a coexistence of the magnetization in magnetic domains with the superconducting state³⁰. On the other hand, the coexistence of superconductivity and antiferromagnetism is much more favorable, since on average a Cooper pair, if the coherence length is large enough, will feel a zero magnetic field. In general, the two phases compete with each other but in some cases there is a coexistence. Competition between antiferromagnetic and superconducting orders is an important characteristic of heavy-fermion systems³¹, which is also shared by high- T_c materials³² and low-dimensional systems³³. Heavy fermion systems that exhibit both superconductivity and antiferromagnetism exhibit ratios between the Néel temperature T_N and the superconducting critical temperature T_c that can vary substantially (of the order of $T_N/T_c \sim 1 - 100$), with coexistence of both types of order below T_c . The coexistence of both types of order can be tuned by external parameters such as externally applied pressure or chemical pressure (involving changes in the stoichiometry)^{31,34}. It has recently been found that UPd_2Al_3 ($T_N = 14.3$ K and $T_c = 2$ K) and UNi_2Al_3 ($T_N = 4.5$ K and $T_c = 1.2$ K) show coexistence of superconductivity and local moment antiferromagnetism^{31,35,36,37,38}. However, in the Ce-based heavy-fermions magnetism typically competes with superconductivity. It has been found in the context of the Anderson model that, both in the problem of local moment formation in the superconductor³⁹ and in the context of the Anderson lattice model, that, in a certain regime, a quantum phase transition is found⁴⁰ which is characterized by an abrupt expulsion of magnetic order by d -wave superconductivity, as an externally applied pressure increases. This transition takes place when the d -wave superconducting critical temperature, T_c , intercepts the magnetic critical temperature, T_m , under increasing pressure.

In our theoretical model, the magnetic moments are considered as a certain magnetic structure embedded into a superconductor. There are different ways to realize it practically by using modern nanotechnology. One of them is related to the formation of a magnetic structure on top of the superconducting layer. As another example, it can be also a laterally organized magnetic superlattice or a hybrid ferromagnet/superconductor structure^{41,42,43,44}.

Various types of heterostructures of superconductors and ferromagnets have been considered in the recent literature^{45,46,47}. Also, the influence of magnetic dots (randomly or regularly distributed) coupled to a superconductor has received attention⁴⁸. In the first case the proximity effect due to the vicinity of the various systems, where both magnetic fluctuations penetrate the superconductor and superconducting fluctuations penetrate the magnetic system⁴⁹, has received particular attention due to potential device applications. The penetration of the magnetic

field in the superconductor splits the up and down spin electron bands, due to the Zeeman effect, in addition to the orbital effect through the vector potential. The Cooper pairs have a finite momentum due to the band splitting and the order parameter oscillates in the superconducting phase. If the size of the superconducting region is small enough, these oscillations have noticeable consequences, such as for instance oscillations in the critical temperature of the superconducting region in F/S/F structures⁴⁵ or varying relative phases of the superconducting wave functions in S/F/S structures if the thickness of the superconducting region is changed. This has been confirmed looking at the Josephson current through the heterostructure⁴⁵. In particular, the so-called π -junctions, where the Josephson current vanishes, have received attention in the literature^{50,51}. In the second case of a distribution of magnetic dots in the close vicinity of the superconductor it has been shown that if the magnetic moments coupled are oriented via an external field, they act as very effective pinning centers for vortices present in the superconductor⁵² originating so called frozen flux superconductors⁵³. The insertion of magnetic rods in the superconductor is also interesting^{54,55,56}.

Now the interest to the superconductivity on a magnetic profile arises on a quite different ground. It was found that electronic properties of magnetic nanostructures can be used in various magnetoelectronic devices, in which the magnetic state can be effectively controlled by the magnetic field or electric current and, in turn, the variation of the magnetic state changes strongly the electronic characteristics of devices (and *vice versa*)^{57,58}. More recently, the semiconducting magnetic materials are included into consideration^{59,60,61,62}, and the superconductors are also used as some elements of the hybrid structures for magnetoelectronic applications⁶³. It seems therefore worthwhile to consider the same type of phenomena in a superconductor with magnetic impurities disposed in some form of ordering that may be controllable from the outside^{64,65,66}. In particular, we have in mind finite systems to which we may attach leads through which we may insert currents that go through a superconducting material with classical spins immersed with their spins oriented in such a way that they form domain walls. These may be achieved for instance imposing different boundary conditions between two sides of the material. Therefore in this work we have in mind systems that are finite and with non-periodic boundary conditions.

While it is interesting to consider the effect the superconducting state may have on the magnetic profile in view of possible spintronics applications, it is also interesting to see the effect of the patterned magnetization profile on the superconducting properties. In this work we focus on the latter aspect of the problem imposing a fixed magnetic pattern. Usually people consider, say, a semiconductor in a potential quantum well or a metal in a magnetic profile but not a superconductor in a magnetic profile. We find that the impurities affect the properties in a very local way and the pattern of interferences between the impurity induced states is rather complex. We also find quantum phase transitions in these situations. Even though we present our results for a fixed magnetic profile, we also study the stability of the magnetic profile taking as stabilizing factor a possible RKKY interaction between the impurities mediated by the quasiparticles of the superconductor.

In section II we introduce the model that describes the magnetic impurities inserted in the BCS *s*-wave superconductor. In section III we consider the quantum phase transitions originated by the change of the spin coupling between the classical impurity spins and the conduction electron spin density, revealed in the structure of the energy levels, local spin density, local gap function and global spin density. In section IV we study the nature of the quasiparticle states revealed in the local density of states and the local kinetic energy. In section V we consider the stability of the domain wall, taking into account an effective interaction between the impurity spins that may originate in a RKKY interaction, and in section VI we study the effect of a finite temperature, particularly on the quantum phase transition. We conclude with section VII.

II. MODEL

Consider a set of classical spins immersed in a two-dimensional *s*-wave conventional superconductor. We consider a two-dimensional system for computational simplicity and because it is easier to experimentally control either the location of the magnetic impurities or the local magnetic fields induced by vicinity of, for instance, magnetic dots. We use a lattice description of the system. In some sites we place classical spins parametrized like

$$\frac{\vec{S}_l}{S} = \cos \varphi_l \vec{e}_x + \sin \varphi_l \vec{e}_z \quad (1)$$

where S is the modulus of the spin. Thus, we assume that the spins lie in the $x - z$ plane. The Hamiltonian of the system is given by

$$\begin{aligned} H = & - \sum_{\langle i,j \rangle, \sigma} t_{i,j} c_{i\sigma}^\dagger c_{j\sigma} - \mu \sum_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} \\ & + \sum_i \left(\Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \Delta_i^* c_{i\downarrow} c_{i\uparrow} \right) \end{aligned}$$

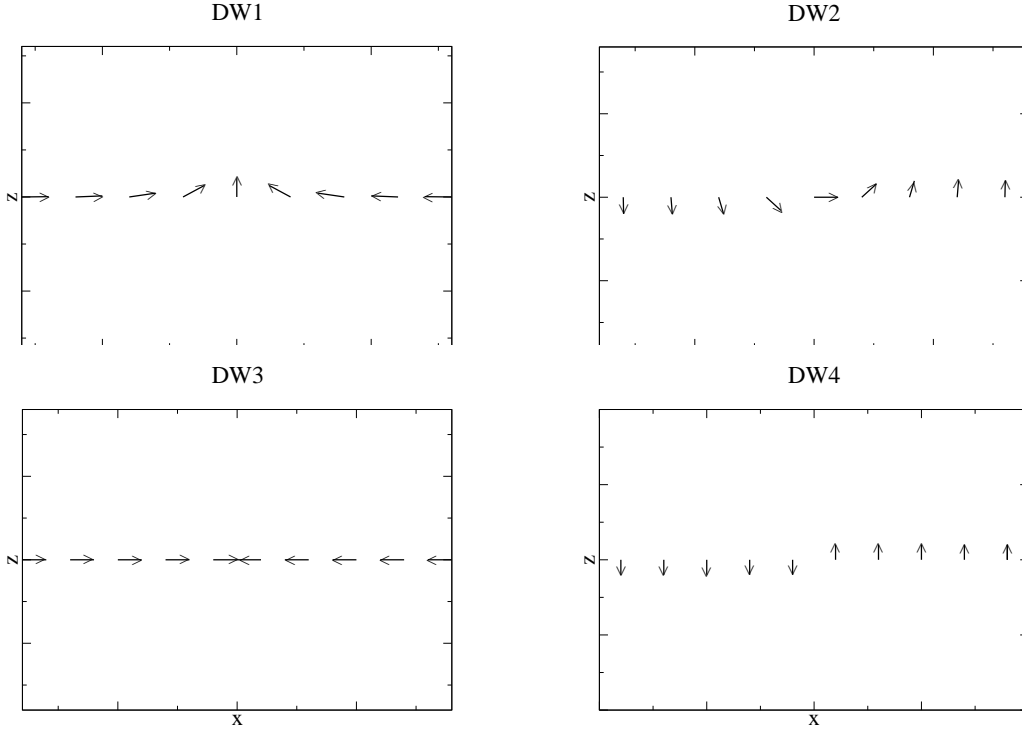


FIG. 1: Various Néel type domain walls considered in this paper.

$$\begin{aligned}
 & - \sum_{i,l,\sigma,\sigma'} J_{i,l} [\cos \varphi_l c_{i\sigma}^\dagger \sigma_{\sigma,\sigma'}^x c_{i\sigma'} \\
 & + \sin \varphi_l c_{i\sigma}^\dagger \sigma_{\sigma,\sigma'}^z c_{i\sigma'}],
 \end{aligned} \tag{2}$$

where the first term describes the hopping of electrons between different sites on the lattice, the second term includes the chemical potential μ , the third one corresponds to the superconducting s -pairing with the site-dependent order parameter Δ_i , and the last term is the exchange interaction of an electron at site i with the magnetic impurity located at site l . The hopping matrix is given by $t_{i,j} = t\delta_{j,i+\delta} + t'\delta_{j,i+\delta'}$ where δ is a vector to a nearest-neighbor site and δ' to a next-nearest site. Most of our calculations will be performed taking $t = 1$, $t' = 0$ and $\mu = -1$. For this value of the chemical potential the band is between quarter and half-filling. The effects of introducing a next-nearest-neighbor hopping or varying the chemical potential are discussed ahead. Note that both the indices l and i, j specify sites on a two-dimensional system. The indices $i, j = 1, \dots, N$, where N is the number of lattice sites. We take $J_{i,l} = J\delta_{i,l}$ and therefore the last sum is over the sites, l , where a spin is located. We assume that the spin configuration is fixed and static. Later on we will study the stability of the spin configuration.

The diagonalization of this Hamiltonian is performed using the Bogoliubov transformation in the form

$$\begin{aligned}
 c_{i\uparrow} &= \sum_n [u_n(i, \uparrow)\gamma_n - v_n^*(i, \uparrow)\gamma_n^\dagger] \\
 c_{i\downarrow} &= \sum_n [u_n(i, \downarrow)\gamma_n + v_n^*(i, \downarrow)\gamma_n^\dagger]
 \end{aligned} \tag{3}$$

Here n is a complete set of states, u_n and v_n are related to the eigenfunctions of Hamiltonian (2), and the new fermionic operators γ_n are the quasiparticle operators. These are chosen such that in term of new operators

$$H = E_g + \sum_n \epsilon_n \gamma_n^\dagger \gamma_n \tag{4}$$

where E_g is the groundstate energy and ϵ_n are the excitation energies. As a consequence

$$\begin{aligned} [H, \gamma_n] &= -\epsilon_n \gamma_n \\ [H, \gamma_n^\dagger] &= \epsilon_n \gamma_n^\dagger \end{aligned} \quad (5)$$

The coefficients $u_n(i, \sigma)$, $v_n(n, \sigma)$ can be obtained solving the Bogoliubov-de Gennes equations⁶⁷. Defining the vector

$$\psi_n(i) = \begin{pmatrix} u_n(i, \uparrow) \\ v_n(i, \downarrow) \\ u_n(i, \downarrow) \\ v_n(i, \uparrow) \end{pmatrix}$$

the BdG equations can be written as

$$\mathcal{H}\psi_n = \epsilon_n \psi_n \quad (6)$$

where the matrix \mathcal{H} at site i is given by

$$\mathcal{H} = \begin{pmatrix} -h - \mu - J_{i,l} \sin \varphi_l & \Delta_i & -J_{i,l} \cos \varphi_l & 0 \\ \Delta_i^* & h + \mu - J_{i,l} \sin \varphi_l & 0 & -J_{i,l} \cos \varphi_l \\ -J_{i,l} \cos \varphi_l & 0 & -h - \mu + J_{i,l} \sin \varphi_l & \Delta_i \\ 0 & -J_{i,l} \cos \varphi_l & \Delta_i^* & h + \mu + J_{i,l} \sin \varphi_l \end{pmatrix}$$

where $h = (t\hat{s}_\delta + t'\hat{s}_{\delta'})$ with $\hat{s}_\delta f(i) = f(i + \delta)$ and $\hat{s}_{\delta'} f(i) = f(i + \delta')$. The solution of these equations gives both the energy eigenvalues and eigenstates. The problem involves the diagonalization of a $(4N) \times (4N)$ matrix. The solution of the BdG equations is performed self-consistently imposing at each iteration that

$$\Delta_i = \frac{V}{2} [\langle c_{i\uparrow} c_{i\downarrow} \rangle - \langle c_{i\downarrow} c_{i\uparrow} \rangle] \quad (7)$$

where V is the effective attractive interaction between the electrons. Using the canonical transformation this can be written as

$$\begin{aligned} \Delta_i &= -V \sum_{n, 0 < \epsilon_n < \hbar\omega_D} \{f_n(u_n(i, \uparrow)v_n^*(i, \downarrow) + u_n(i, \downarrow)v_n^*(i, \uparrow)) \\ &\quad - \frac{1}{2}[u_n(i, \uparrow)v_n^*(i, \downarrow) + u_n(i, \downarrow)v_n^*(i, \uparrow)]\} \end{aligned} \quad (8)$$

where ω_D is the Debye frequency, and f_n is the Fermi function defined as usual like

$$f_n = \frac{1}{e^{\epsilon_n/T} + 1}$$

where T is the temperature. We note that the Bogoliubov-de Gennes equations are invariant under the substitutions $\epsilon_n \rightarrow -\epsilon_n$, $u(\uparrow) \rightarrow v(\uparrow)$, $v(\uparrow) \rightarrow u(\uparrow)$, $v(\downarrow) \rightarrow -u(\downarrow)$, $u(\downarrow) \rightarrow -v(\downarrow)$.

We are interested in calculating various quantities. In particular we calculate the quasiparticle spin densities

$$\begin{aligned} s_z(i) &= \frac{1}{2} \langle c_{i,\sigma}^\dagger \sigma_{\sigma,\sigma'}^z c_{i,\sigma'} \rangle \\ &= \frac{1}{2} \sum_n \{f_n[|u_n(i, \uparrow)|^2 - |u_n(i, \downarrow)|^2] \\ &\quad + (1 - f_n)[|v_n(i, \uparrow)|^2 - |v_n(i, \downarrow)|^2]\} \\ s_x(i) &= \frac{1}{2} \langle c_{i,\sigma}^\dagger \sigma_{\sigma,\sigma'}^x c_{i,\sigma'} \rangle \\ &= \frac{1}{2} \sum_n \{f_n[u_n^*(i, \uparrow)u_n(i, \downarrow) + u_n^*(i, \downarrow)u_n(i, \uparrow)] \\ &\quad - (1 - f_n)[v_n^*(i, \uparrow)v_n(i, \downarrow) + v_n^*(i, \downarrow)v_n(i, \uparrow)]\} \end{aligned} \quad (9)$$

where the sums are taken over the positive excitation energies. Also, we are interested in calculating the local density of states

$$\begin{aligned} \rho(\epsilon, i) &= \sum_{n,\sigma} \{|u_n(i, \sigma)|^2 \delta(\epsilon - \epsilon_n) \\ &\quad + |v_n(i, \sigma)|^2 \delta(\epsilon + \epsilon_n)\} \end{aligned} \quad (10)$$

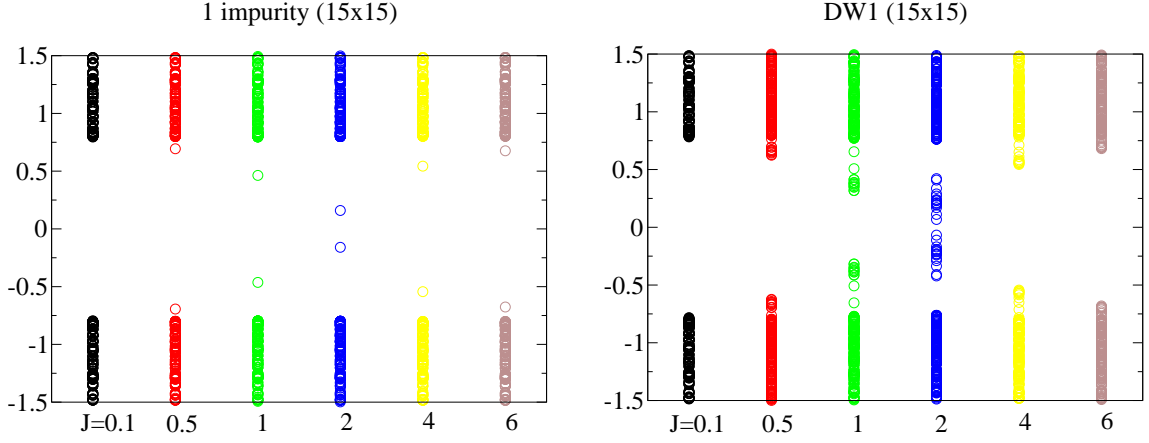


FIG. 2: a) Energy levels for 1 impurity. There is one pair of boundstates originated by the impurity. As J grows the level appears to "cross" zero energy. The crossing is between $J = 1$ and $J = 2$ (see ahead). b) Energy levels for DW1 domain wall.

Since the system is finite the states are discrete. This can be written as

$$\rho(\epsilon, i) = \sum_{n, \sigma, \alpha} \rho_{\alpha}(\epsilon_n, i, \sigma) \quad (11)$$

where $\alpha = +$ runs over the positive energy eigenvalues of Eq. (6) ϵ_n^+ and $\alpha = -$ runs over the negative energies ϵ_n^- . Therefore

$$\rho_+(\epsilon_n, i, \sigma) = |u_n(i, \sigma)|^2 \delta_{\epsilon, \epsilon_n^+} + |v_n(i, \sigma)|^2 \delta_{\epsilon, \epsilon_n^-} \quad (12)$$

and

$$\rho_-(\epsilon_n, i, \sigma) = |v_n(i, \sigma)|^2 \delta_{\epsilon, \epsilon_n^+} + |u_n(i, \sigma)|^2 \delta_{\epsilon, \epsilon_n^-} \quad (13)$$

where $\epsilon_n^{\alpha} = -\epsilon_n^{-\alpha}$.

Consider now the possibility that there is a line of impurities, \vec{S}_l , such that their spin orientations are correlated. For simplicity we consider a one-dimensional array of spins that traverse the two-dimensional material from one border to the other along the x direction. A dense impurity distribution will destroy superconductivity if the spin coupling is strong enough. Therefore, to study the effects of interference we limit ourselves to lines of spins in the two-dimensional electronic system. One may impose a magnetic field at each side of the sample such that it orients the first and last spins. Choosing different boundary conditions we may create different domain walls assuming that there are interactions between the spins that tend to orient them. These interactions may either be ferromagnetic or antiferromagnetic and may have its origin in RKKY type interactions via the superconducting substrate. As we mentioned before, the form of the interactions mediated by the quasiparticles has a form similar to the one of a standard metal.

Clearly if the number of spins increases considerably and/or their coupling to the electron density increases enough superconductivity will be destroyed. We will be focusing on situations where superconductivity prevails, as evidenced by the self-consistent solution of the BdG equations. In particular, if we consider a fully two-dimensional distribution of classical spins, unless the coupling is small, in general superconductivity will be destroyed. Since the Zeeman term of each classical spin acts like a local magnetic field we expect at the very least that if the applied field exceeds the critical field superconductivity will be destroyed.

The various cases considered are displayed in Fig. 1. Besides the cases of one impurity and two impurities previously considered, and that we briefly consider here to compare with the new results, we consider various situations where for instance, we have a domain wall of the Néel type (here limited to a line of spins to simplify) where the leftmost spin is either oriented along the chain (defined as the x direction) or perpendicularly to it (these are the cases *DW1* and *DW2*, respectively). Specifically in the case of domain wall *DW1* we choose

$$\varphi_l = \frac{\pi}{2} + \frac{\pi}{2} \tanh \frac{x - x_c}{\lambda} \quad (14)$$

for the domain wall *DW2*

$$\varphi_l = \frac{\pi}{2} \tanh \frac{x - x_c}{\lambda} \quad (15)$$

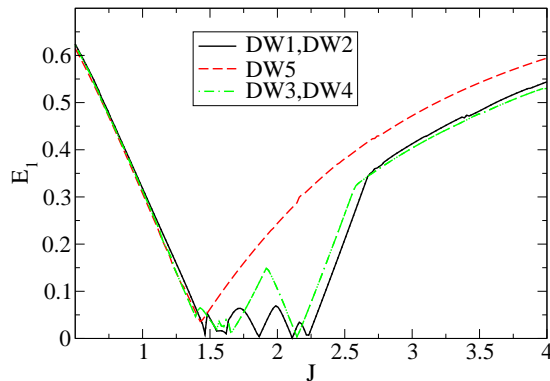


FIG. 3: Lowest energy level as a function of J for the various domain walls. Note the various level crossings (near zeros) for the various cases. The crossings are concentrated for $1 < J < 2$. Note that for the antiferromagnetic chain there is a single level crossing. Domain walls DW1 and DW2 have the same lowest state energy and the same happens with domain walls DW3 and DW4.

The other configurations are characterized by $\varphi_l = \frac{\pi}{2}\theta(-x) - \frac{\pi}{2}\theta(x)$ for the domain wall DW3, $\varphi_l = \pi\theta(-x)$ and $\varphi_l = 0$ for $x > 0$, for DW4 and $\varphi_l = \frac{\pi}{2}(1 + (-1)^x)$, for the antiferromagnetic case (AF), DW5,

Most of our results were obtained for a system of 15×15 sites. Increasing the system size does not affect qualitatively the results. For instance, we have considered a system of 25×25 sites and the results are very similar. The only visible difference is the reduction of the finite size effects near the borders of the system. We used parameters such that superconductivity is stable and the superconducting gap is relatively large so that the in-gap states are easily identified. Choosing units where the hopping $t = 1$, we get a gap of the order of 0.4 choosing $V = 4t$. Typically we choose $\lambda = 3$.

III. QUANTUM PHASE TRANSITIONS

A. Energy levels

We begin by revisiting some results for a situation where we have one or two impurities immersed in the material. As is typical of a s -wave pairing the clean system has a gap. Introducing one magnetic impurity generates two bound states in the gap. One is at positive energies and the other is at a symmetric negative energy. This is clearly shown in Fig. 2. The nature of the wavefunctions corresponding to the two eigenstates will be discussed later. If we introduce two impurities the number of states in the gap doubles. In general if there are N_i impurities there are $2N_i$ states in the gap, N_i at positive energies and the same at negative energies. As the coupling between the electron spin density and the impurity spin grows, the energy of the boundstate lowers and approaches the Fermi level. Increasing further the coupling the level is repelled from the chemical potential. The nature of the groundstate has changed, as we will see ahead.

In Fig. 2 we also show the energy spectra for the domain wall DW1 for different values of the coupling. There are now as many boundstates as impurity spins (for positive energies). For small coupling the energy levels are close to the top of the gap, but as the coupling grows the trend is similar to the single impurity case. For a large system the gap will be virtually filled close to the transition where the level crossing(s) take place. As we will see there are several level crossings as the coupling varies. Note that J actually means JS where S is the magnitude of the local spins. Therefore we may consider $S = 1/2$ and change the Zeeman term coupling value. But we can as well consider that the coupling is not changing much but we are inserting impurity spins with different values of S . This is truly the classical limit. Previously the effect of the change in the relative angles or distances between two impurities was considered²⁰. These changes also induce changes in the energy spectra and affect the various level crossings. The corresponding change in relative angles and/or distances between two impurities in the case of domain walls is the change in the width of the domain wall, λ . However, the analysis of these interference effects on the states is more complex due to the multiple scatterings, as we will show ahead.

In contrast to the case of a small number of isolated magnetic moments creating a number of levels in the gap, an ordered array of impurities creates some minibands. It can be clearly seen in Fig. 2, even though we performed our calculations for a finite system.

The detail of the level crossings is better studied considering the evolution of the lowest level (with positive energy)

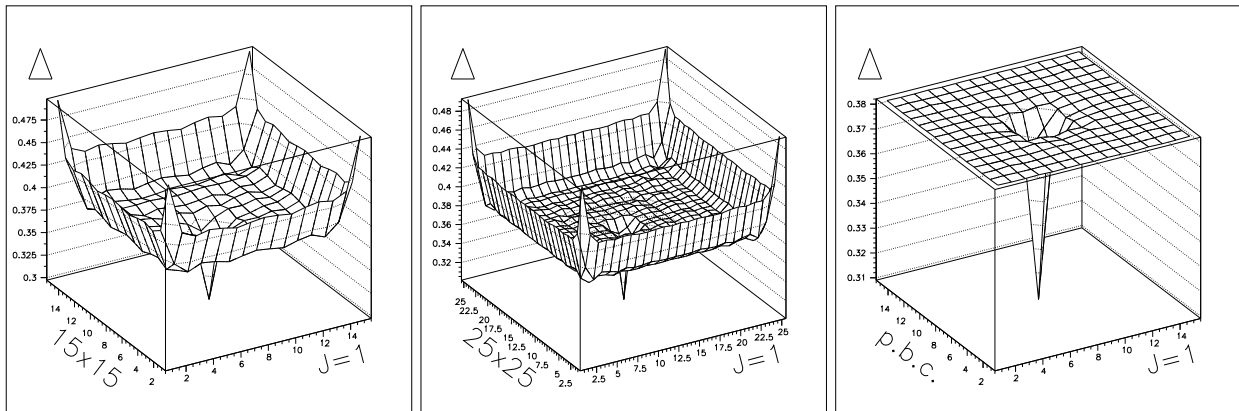


FIG. 4: Comparison of the spatial dependence of the gap function for the case of a single impurity, for the cases a) 15×15 and b) 25×25 for open boundary conditions and c) periodic boundary conditions, for $J = 1$.

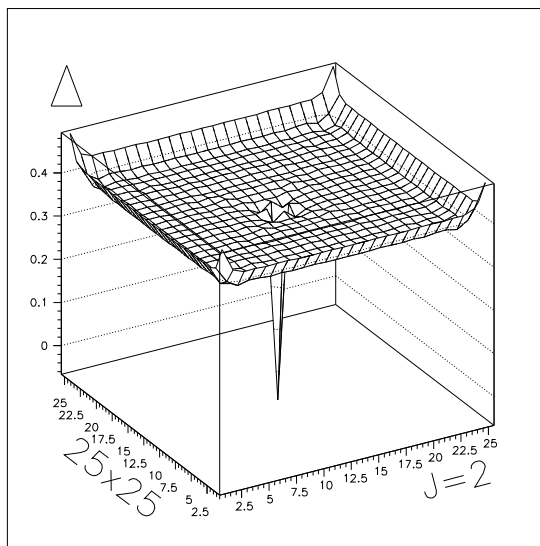


FIG. 5: Δ for $J = 2$ for one impurity. Δ decreases at impurity site (this is the same as for a non-magnetic impurity). Note that there is a π shift in Δ (it becomes negative) at the impurity site in contrast to the case for $J = 1$, shown in Fig. (4b).

as a function of the coupling, J . This is shown in Fig. 3 for various systems. In the case of a single impurity there is a single level crossing, which for our parameters occurs at a value between $J = 1$ and $J = 2$. In the case of the domain walls there are several level crossings, as illustrated in Fig. 3. For the same set of parameters the level crossings are contained in an interval of coupling strengths that is of the same order. We recall that at each level crossing the lowest energy does not strictly reach zero. At a finite value of the coupling the crossing occurs without closing the gap indicating that the quantum phase transition is actually a first order one. The same occurs in the cases of the domain walls. However, since the number of boundstates inside the gap increases, the mini-gaps near the various transitions are quite small and in the thermodynamic limit should become vanishingly small. Even for a small system, considering, for instance, a two-dimensional distribution of impurity spins, one expects that the gap will be filled with states. Actually, for a moderate coupling of the order of $J \sim 1$, the superconductivity can be destroyed by the local magnetic fields created by the impurities. The states in the gap are localized as will be shown later.

B. Local gap function

A characteristic of the order in the system is given by the behavior of the superconducting order parameter, as a function of space⁶⁸. In Fig. 4 we study the effects of system size and boundary conditions on the gap function in the case of a single impurity, located at the center of the system. We see that for open boundary conditions the finite size

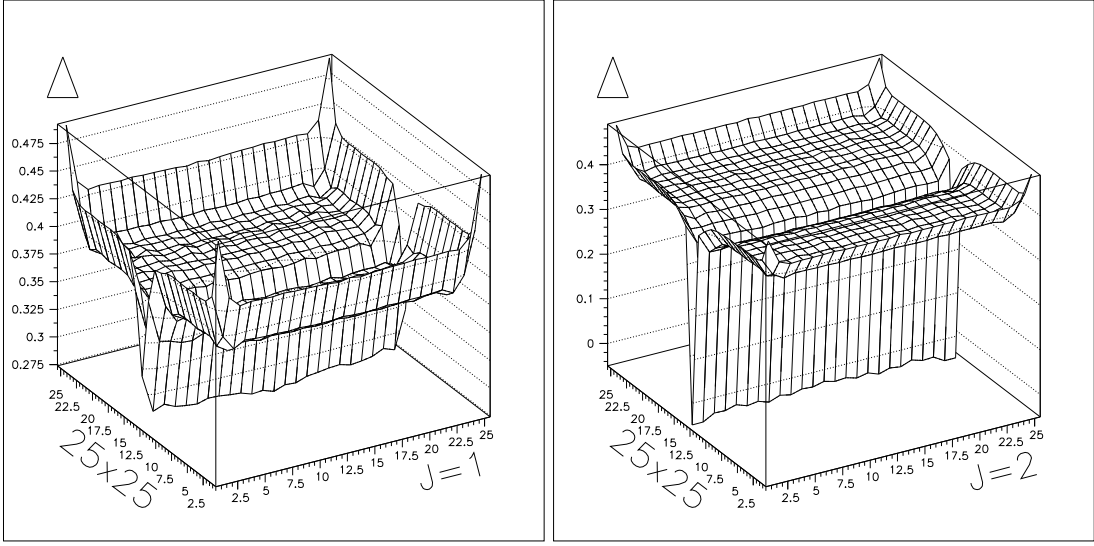


FIG. 6: Δ for DW1 for $J = 1, 2$. Note that for $J = 2$ there is again a π shift.

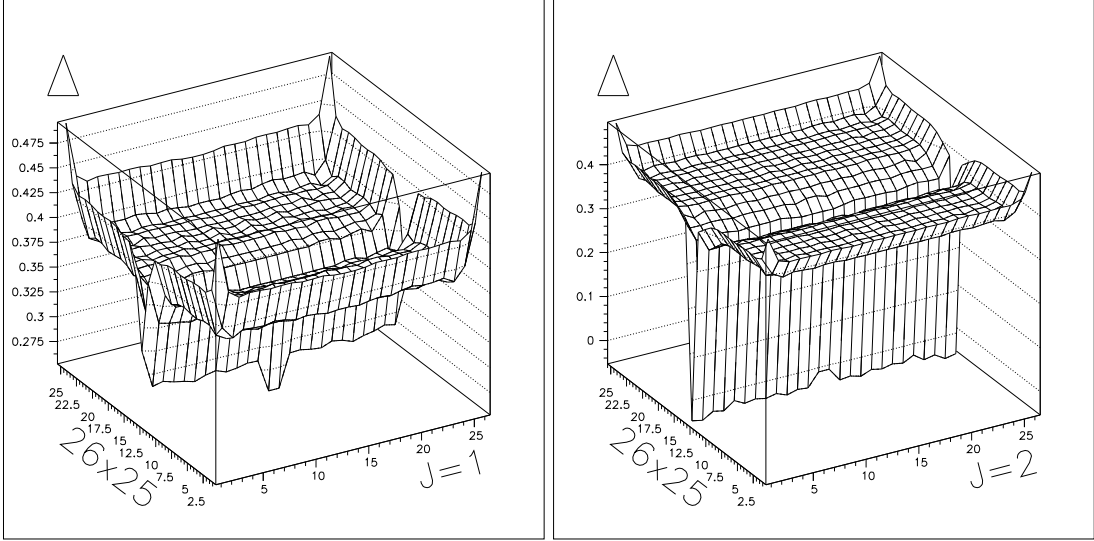


FIG. 7: Δ for DW4 for $J = 1, 2$. Once again there is a π shift.

effects are important near the border of the system. Increasing the system size these diminish. In the case of periodic boundary conditions the finite size effects are virtually absent. Note that away from the borders the results for the cases 15×15 and 25×25 are qualitatively the same. Therefore in the rest of the paper we will illustrate the results considering either system sizes preferring however the smaller system size since this decreases the computational time. Also, as discussed above, we are aiming at the equilibrium properties of a system which is finite, and therefore we will consider the open boundary conditions instead of the more standard periodic boundary conditions.

In Fig. 5 we compare Δ for the typical couplings of $J = 1$ and $J = 2$ for the case of one impurity. The first thing to notice is that the order parameter is only affected very close to the impurity site. Since we are using open boundary conditions the behavior of the order parameter, and other quantities is also affected near the border. If we were to use periodic boundary conditions these effects would be strongly reduced and for a relatively large system they would be almost vanishing. We note however that near the impurity the boundary conditions have no effect as expected. We note the previously observed π shift of the order parameter when J is large enough. This effect prevails when we have several impurities. We see from Figs. 6, 7 that the π shift is observed for the more complex structures, indicating once again that some properties are of a local nature. Also we see that the orientation of the spins does not affect significantly the order parameter. This is to be expected since we are considering singlet pairing, and therefore rotationally invariant. While the spin density is obviously strongly dependent on the impurity spin orientation, the

order parameter is only mildly affected particularly at the center of the solitonic like spin configuration. If the coupling is strong enough the order parameter is basically constant along the impurity line.

As the level crossings occur there is a change of phase of π locally, as evidenced by the single impurity results. Therefore, as the various level crossings occur in the cases of several impurities there may be inhomogeneities since the capture of the electrons by the impurities is not necessarily uniform, as evidenced by the non-homogeneous changes of local spin density. Therefore it is natural to expect some inhomogeneities along the chain. This is also seen in the LDOS shown ahead.

A simple explanation for the occurrence of the π shift is not available¹. It is argued that it is related to the π -junctions referred above, but these occur for specially commensurate widths of the ferromagnetic slab in /S/F/S heterostructures. Here the effect is quite local and therefore a relation is not clear.

C. Local spin density

In Fig. 8 (a) and (b) we show the behavior of the electron spin density in the presence of two impurities. Since the impurity spin acts like a local magnetic field we expect that the spin density will align along the local field. At the impurity site the spin density is aligned as shown in the figure. In Fig. 8 we compare two values of the coupling. For $J = 1$ note the negative spin density around the impurity site. At the impurity site it is positive, as expected. For larger J such as $J = 2$ notice that the spin density in the vicinity of the impurity site is now positive. We will see that for $J = 1$ the total $s_z = 0$, while for larger values of J it is positive (this is one of the hallmarks of the phase transition). One interpretation is that if J is strong enough the impurity captures one electron breaking a Cooper pair leaving the other electron unpaired and the overall electronic system becomes polarized. At zero temperature, where the quantum phase transition occurs, the magnetization reduces to

$$\begin{aligned} s_z(i) &= \frac{1}{2} \sum_{n, \epsilon_n > 0} (|v_n(i, \uparrow)|^2 - |v_n(i, \downarrow)|^2) \\ &= \frac{1}{2} \sum_{n, \epsilon_n < 0} (|u_n(i, \uparrow)|^2 - |u_n(i, \downarrow)|^2) \end{aligned} \quad (16)$$

and therefore may be calculated either from the hole states at positive energies (given by v_n) or by the particle states at negative energies (given by u_n). At zero temperature there are no quasiparticles and therefore all the states of positive energies are empty. We will see later the nature of the states as a function of coupling and their relation to the spin orientation.

In Fig. 8 (c)-(f) we show the results for the spin density for two typical cases, *DW1* and *DW4*, for the same couplings. For $J = 1$ the first level crossing has not yet occurred and the total magnetization is zero, as for the single impurity case. This implies that the system responds to the local polarization of the electronic spin density at the impurity site by an overall negative polarization, to yield a total magnetization that vanishes. Since the effect of the impurities is quite local, it is around the impurity line that the magnetization is negative. This happens for both domain walls. Increasing the coupling, for instance $J = 2$ the situation changes. It is clear from the figures that around the impurity line the magnetization is no longer negative. The influence of the impurity spins is now strong enough to polarize the electronic system, not only at the impurity sites but also in their vicinity. The total magnetization no longer vanishes but has a finite value. This is further discussed ahead.

As the coupling changes, in general the magnetization varies in a continuous way. However, as the system goes through the various phase transitions the spin density changes discontinuously consistently with a first order quantum phase transition. Since there are now several spins that may bind electrons in succession, there is now a sequence of phase transitions. These changes are not in general uniform along the line of impurities. For instance in the case of the domain wall *DW1* at the first transition (which, for the parameters chosen, occurs near $J = 1.55$) there is an increase centered around the middle point of the chain, for the next transition (around $J = 1.62$) the increase occurs both at the middle point and at both ends of the chain, at the next transition (around $J = 1.8475$) there is a sharp peak at the middle point and at the transition around $J = 2.215$ the increase has a broad maximum centered around the middle point. This inhomogeneity is characteristic of the various domain walls where the space distribution of the wave functions is complex due to the multiple interferences of the quasiparticles off the various impurities.

D. Global spin density

The sequence of the quantum phase transitions is also clearly displayed if we consider the total magnetization of the system as a function of the coupling. In the cases of the domain walls *DW1* and *DW2* the total value of s_z^T and

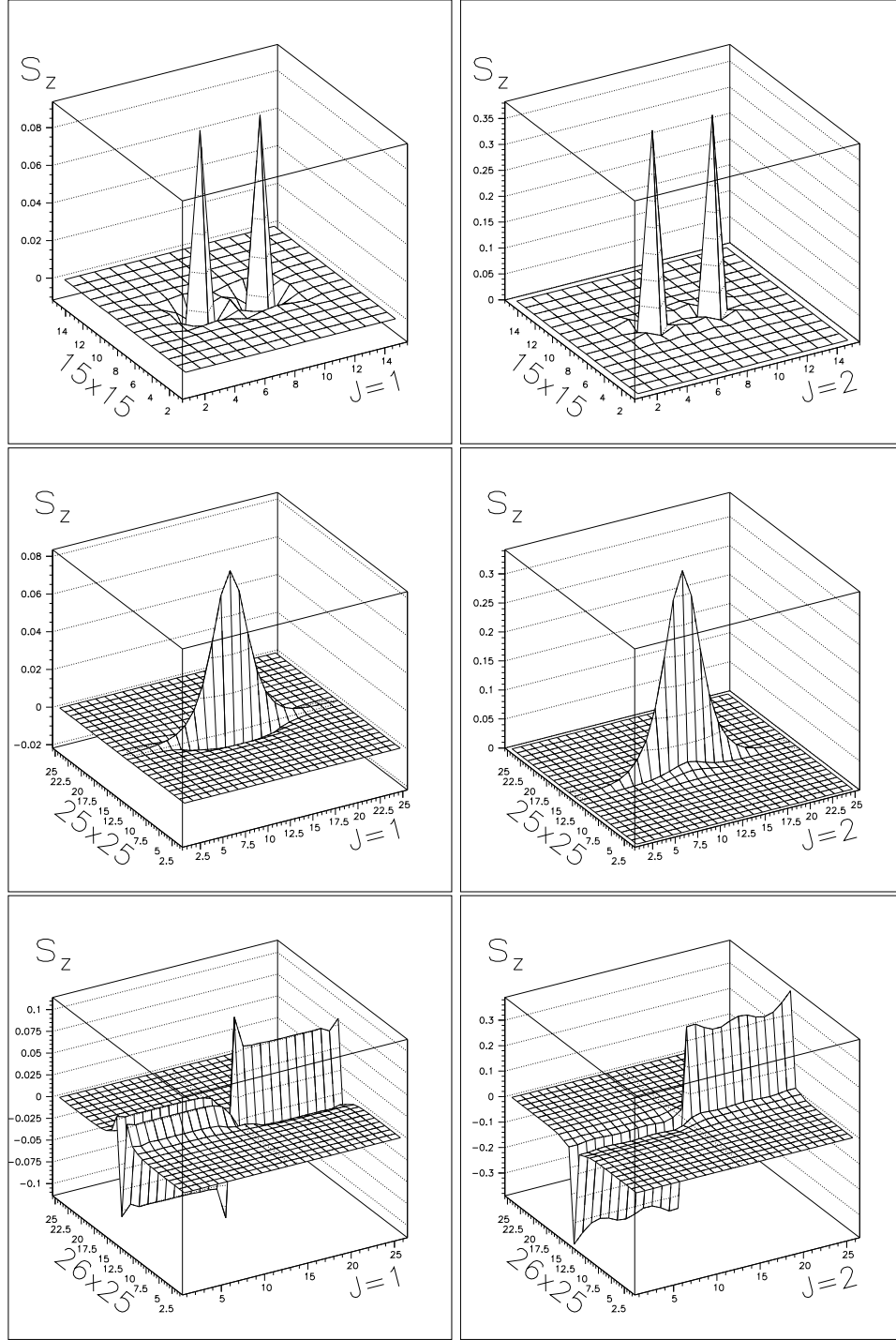


FIG. 8: (a) and (b) Plots of s_z the local z component of the electronic spin density for the case of two impurities for $J = 1, 2$. (c) and (d) Plot of s_z for $J = 1, 2$ for DW1. Note that along the line of spins constituting the domain wall the spin density is large and positive and that for $J = 1$ in the neighbors it is negative and for the other case $J = 2$ it is positive. As for the cases of one and two impurities this transition is associated with a transition from $s_z^T = 0$ to a $s_z^T \neq 0$. (e) and (f) Plot of s_z for $J = 1, 2$ for DW4. Note that now the total magnetizations are zero but if we divide the system in two halves similar considerations apply.

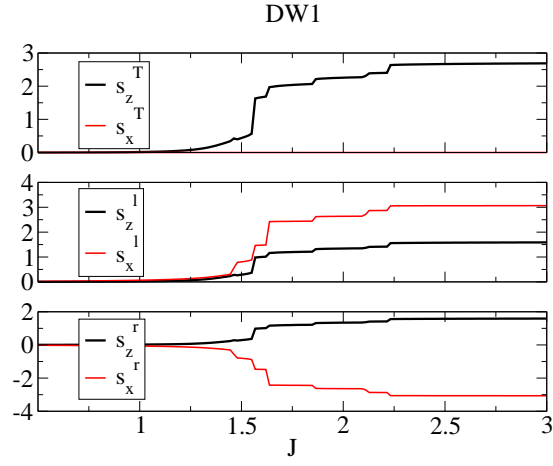


FIG. 9: Various quantities for DW1 as a function of J . We plot s_z^T , s_x^T , s_z^l , s_x^l , s_z^r and s_x^r . Here l and r stand for left half of the system and right half of the system. Due to the symmetry of the domain wall chosen $s_x^T = 0$. Note however that if we consider the magnetization of the left or right halves there are interesting phase transitions.

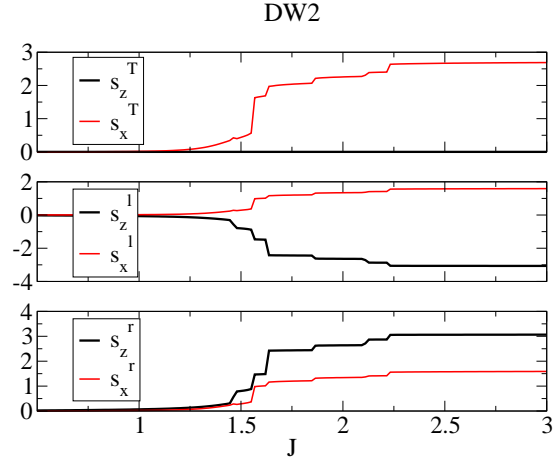


FIG. 10: Various quantities for DW2 as a function of J . We plot s_z^T , s_x^T , s_z^l , s_x^l , s_z^r and s_x^r .

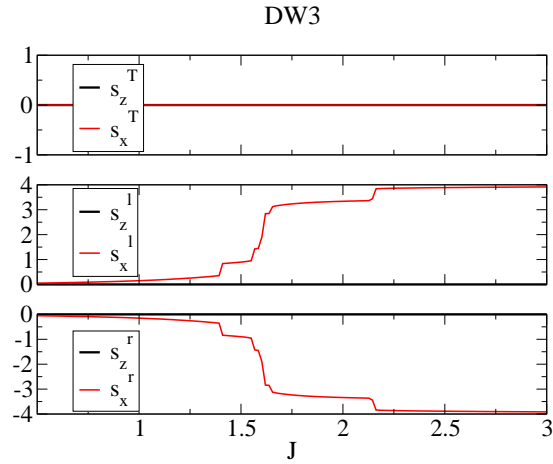


FIG. 11: Various quantities for DW3 as a function of J . We plot s_z^T , s_x^T , s_z^l , s_x^l , s_z^r and s_x^r .

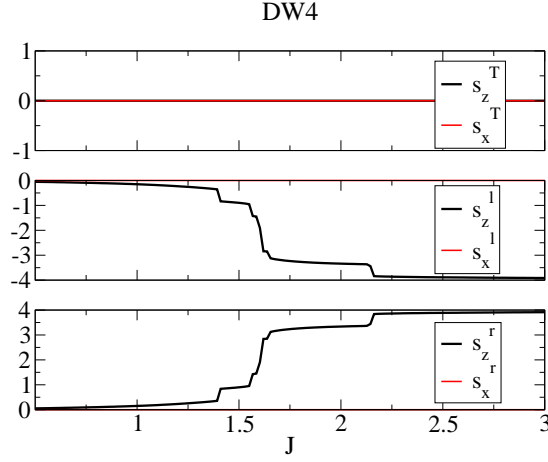


FIG. 12: Various quantities for DW4 as a function of J . We plot s_z^T , s_x^T , s_z^l , s_x^l , s_z^r and s_x^r .

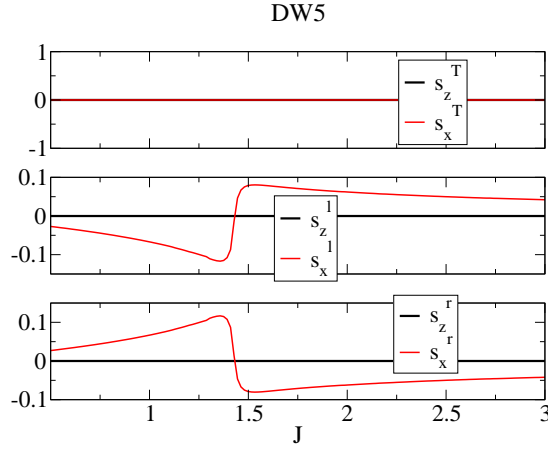


FIG. 13: Various quantities for the AF spin configuration (DW5) as a function of J . We plot s_z^T , s_x^T , s_z^l , s_x^l , s_z^r and s_x^r .

s_x^T changes as a function of the coupling in basically the same way since one may obtain one case from the other by a rotation in spin space. However, the other domain walls considered have no overall magnetization in any direction. We may however consider only the left or right magnetizations and these will display the same type of quantum phase transitions, even though the total magnetizations are in principle zero.

The detail of the dependence on the coupling value is better shown in Figs. 9,10,13,11,12 where we present various average spin densities for the various domain walls. Due to the shape of the domain walls, in some cases the total spin densities, summed over all electron sites, vanish. However, the quantum phase transitions are clearly shown if we take averages over, say, half of the system. Associated with the various level crossings there are various phase transitions between plateaus corresponding to an increasing spin density component as the electron spins bind to the impurity spins.

The antiferromagnetic case is peculiar. There are no true discontinuities but the left and right s_x do not vanish and show an interesting change of sign at a value of $J \sim 1.4$. This is probably a finite size effect that should disappear as the system size increases.

E. Effect of changing the Fermi level and effect of next-nearest-neighbor hopping

We have chosen to work with a chemical potential of $\mu = -1$. The electron density is therefore not fixed and has to be determined self-consistently. We do not present here the results for the electron density but will consider them in future publications, particularly in the context of transport properties where the density of charge carriers will play an important role. The results do not depend in a significant way on the value of the chemical potential. The value of $\mu = -2$ has been chosen before since the system in this case is quarter-filled and the analysis of the interference

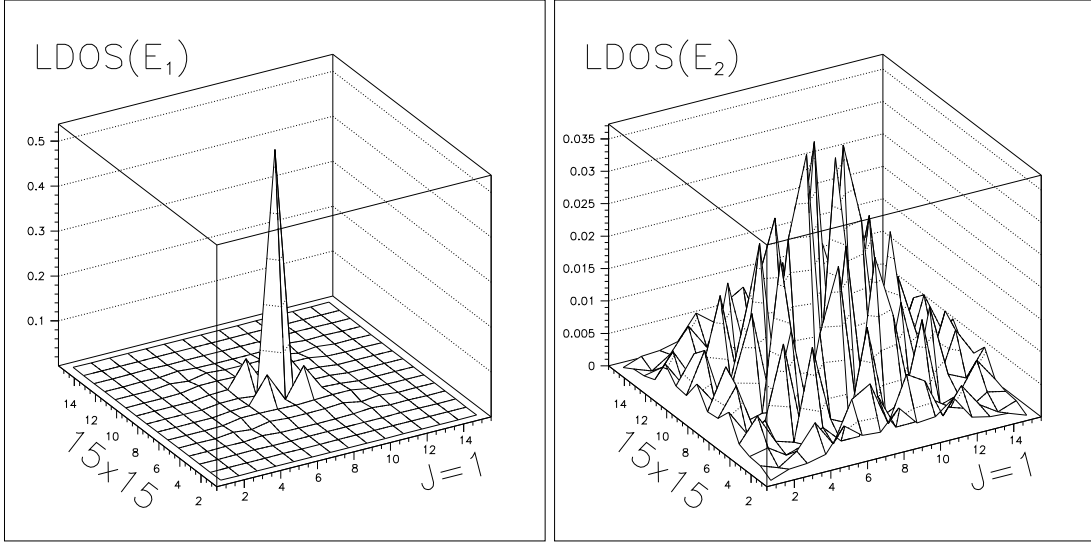


FIG. 14: LDOS for the lowest energy state (with positive energy) and the second lowest state (also with positive energy) for $J = 1$ in the case of a single impurity.

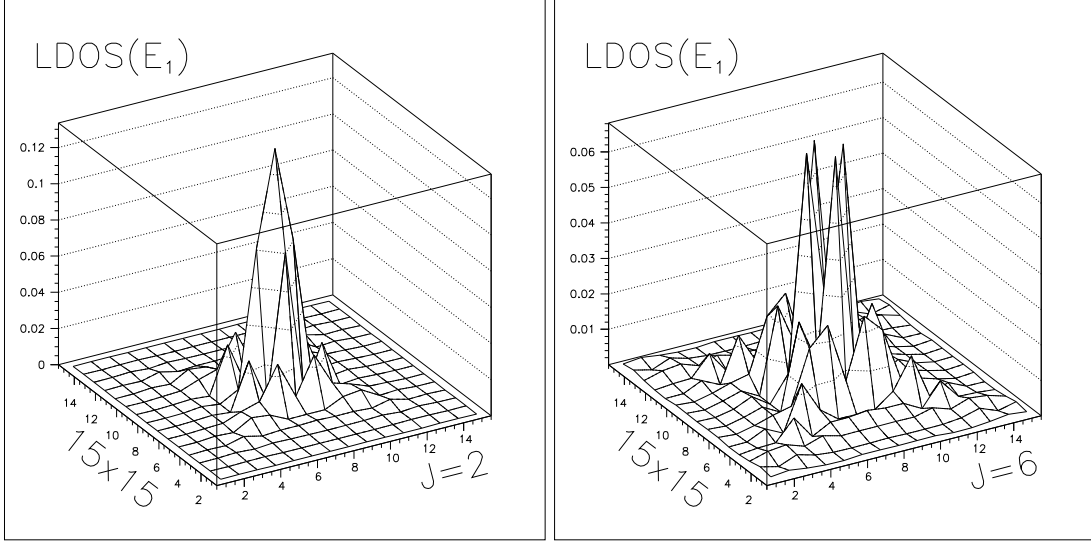


FIG. 15: LDOS for the lowest energy state (with positive energy) for $J = 2, 6$ in the case of a single impurity.

pattern for the two impurities case is simplified²¹, since along the x axis the Fermi momentum is $\pi/2$. In the case of this paper $\mu = -1$ means the band-filling is larger but is of the order of 0.65 and therefore is still quite far from half-filling. The change in the chemical potential slightly affects the location of the quantum phase transitions but the results are qualitatively the same. However, changing the chemical potential affects the band-filling. The band-filling is also affected by the spin coupling. As the coupling grows the band-filling increases since electrons are trapped by the impurities. We note that if we consider a more realistic situation, where we take into account next-nearest-neighbor hopping, the results are also qualitatively the same. The effect of this extra term is to change the location of the quantum phase transition points. Both decreasing the band-filling and introducing the next-nearest-neighbor hopping decrease the order parameter and therefore increase in proportion the importance of the coupling between the spin density and the impurity spin anticipating the appearance of the quantum phase transition.

We note that considering the case of a more dense impurity spin distribution will both tend to destroy superconductivity for smaller values of the coupling but also to increase the band filling approaching the half-filling situation for moderate values of the coupling. This effect will be considered elsewhere.

IV. NATURE OF STATES

A. LDOS: $\rho(\epsilon, i)$

As mentioned before, since the spectrum is discrete and symmetric, the density of states is composed by a series of delta function peaks where for each energy (for instance positive) there are two contributions one from a level of positive energy, for instance ϵ_n , with weight $|u_n(i, \sigma)|^2$ and another from the symmetric energy level with energy $-\epsilon_n$ with weight $|v_{\bar{n}}(i, \sigma)|^2$, where \bar{n} is the level symmetric to the level n .

Consider first the case of a single impurity. The true nature of the boundstate of positive energy is better evidenced by the LDOS at the energy of the lowest level. Specifically, and calling this energy level $n = 1$, the LDOS is given by

$$\begin{aligned} \rho(\epsilon = \epsilon_1, i) &= \sum_{\sigma} (|u_1(i, \sigma)|^2 + |v_{\bar{1}}(i, \sigma)|^2) \\ &= \rho_+(\epsilon_1, i, \uparrow) + \rho_+(\epsilon_1, i, \downarrow) \end{aligned} \quad (17)$$

This is shown in Fig. 14 where the LDOS of the second level (located in the continuum) is also shown for comparison. The LDOS of the second level is spread over the system. The lowest state is localized near the impurity site. The LDOS of the lowest peak is nicely confined around the impurity site and has a large spectral weight. Here we considered $J = 1$. Notice that the oscillations are strongly damped beyond the nearest- neighbors. In Fig. 15 we consider higher values of the coupling $J = 2, 6$. Notice that for $J = 2$ the peak has broadened but is still quite localized. Also, the oscillations are now less damped particularly along the diagonals of the square. Note also that the spectral weight at the impurity site is now reduced. In the case of a large coupling $J = 6$ the peak is still localized but has several new features. It has a broader spectral weight and there is no peak at the central point. At the impurity site the gap function is still negative even though its magnitude decreases for this large J value. Instead of a central peak there are now four peaks in the nearest neighbors of the impurity site and a square symmetry, even though the state is still quite localized. Note the extension of the wavefunctions along the diagonals of the system.

If we consider the case of two impurities there are now two boundstates. The behavior is quite similar to the case of a single impurity with two maxima localized in the vicinity of the impurity sites. The third level and beyond are extended, as expected.

In the cases of a few impurities, like two or three impurities, a study of the relation between the phase transitions and the level structure is possible²¹. However, increasing the number of impurity spins this analysis is complicated by the multiple interferences of the wave functions of the boundstates. We may however look at the LDOS for the lowest levels and examine their structure. This is shown in Figs. 16,17 for the domain wall DW3 for the couplings $J = 1, 2$.

The case of $J = 1$ is similar to the case of a small coupling, where no phase transition has occurred, and the local magnetic field is a small perturbation. Clearly as the coupling increases the shielding of the perturbation in the vicinity of the impurity site increases but no qualitative change is observed until the first level crossing. There are now as many boundstates as impurities and therefore there are several "localized" states. In Fig. 16 we show the LDOS for the first seven positive energy levels and for a level in the continuum. The states in the gap have a structure along the line of impurities and, in that sense, they are localized. However, since there are several impurities there is a series of maxima and minima at the impurity sites. In the single impurity or two impurities cases the boundstates have maxima at the impurity locations but we see from Fig. 16 that this is not so. For instance, the lowest level wavefunction has a series of peaks at alternating sites symmetrically around the central point (note that the number of sites is odd). On the other hand the second level has a zero at the central point indicative of an anti-symmetric state. This is reminiscent of the results obtained for two impurities²¹ where there is naturally two states, one symmetric and one anti-symmetric. On the other hand the third peak is quite spread along the domain wall, and extends further along the perpendicular direction. As the energy increases the states are still fairly localized with more or less complex structures. As before, the state in the continuum is of a different nature and spread through the system, even though not homogeneously. The spectral weights are similar even though their magnitudes are correlated inversely with their extension, as expected.

Increasing the coupling to $J = 2$ changes somewhat the wave-functions. The first levels are still localized but with different spatial distributions. For instance the two lowest levels display two broad peaks and deep minima at the central position of the domain wall. The third level is somewhat similar to the case of $J = 1$. Also, we see an alternancy of symmetric and anti-symmetric states as the energy increases.

We should note that the results obtained for the 15×15 system are not general. For instance, considering the case of a system 25×25 the details of the LDOS are different. In particular, due to the increased number of boundstates the sequence of states is more complex. Also, the increased number of states in the gap increases the near degeneracies of the states and mixes their symmetry properties. As discussed above, as the spin coupling grows the impurities

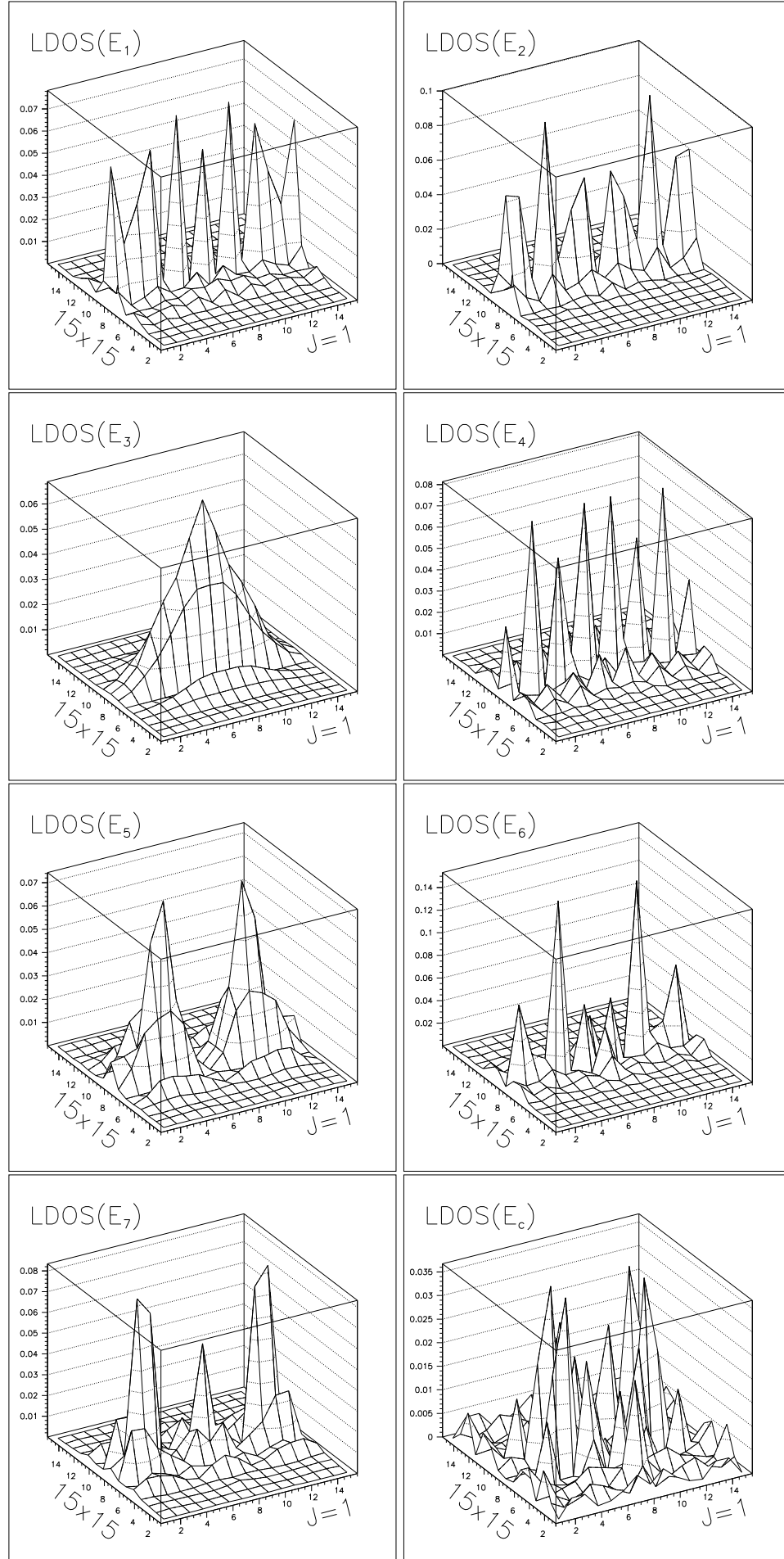


FIG. 16: LDOS for DW1 for the first seven levels for $J = 1$ and a state in the continuum.

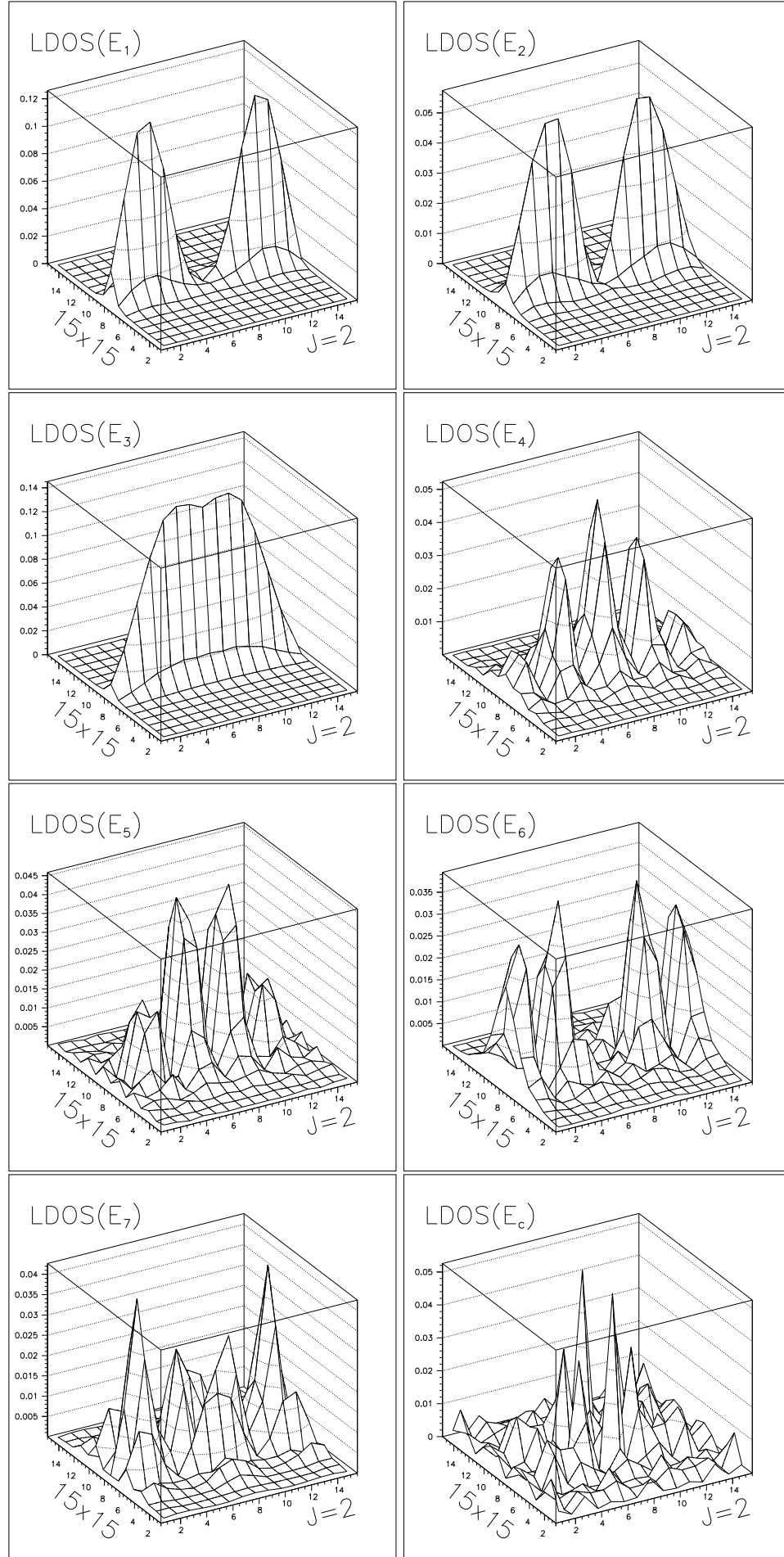


FIG. 17: LDOS for DW1 for the first seven levels for $J = 2$ and a state in the continuum.

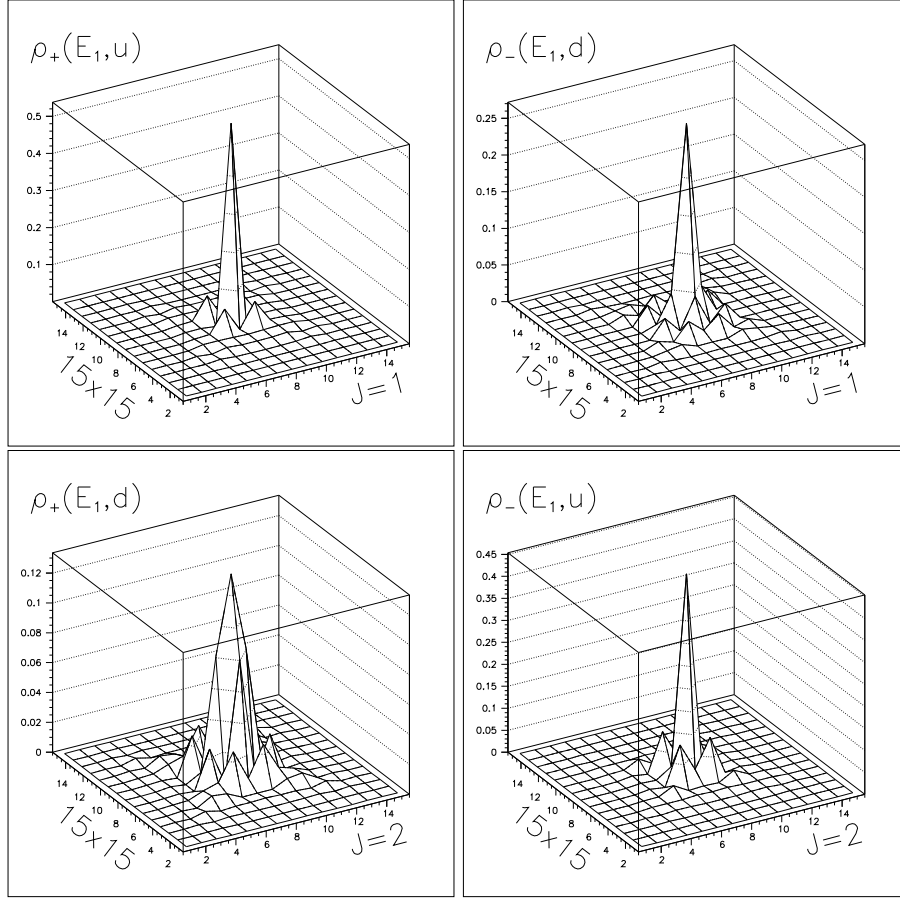


FIG. 18: LDOS for a single impurity. a) $\rho_+(\epsilon_1, i, \uparrow)$ for $J = 1$, b) $\rho_-(\epsilon_1, i, \downarrow)$ for $J = 1$, c) $\rho_+(\epsilon_1, i, \downarrow)$ for $J = 2$, d) $\rho_-(\epsilon_1, i, \uparrow)$ for $J = 2$. (Note that in the vertical axis label $u = \uparrow$ and $d = \downarrow$).

capture electrons. In the case of the extended spin configurations, since the states are extended along the chain of spins, even though the lowest states are localized in the perpendicular direction, it is not possible to disentangle the capture of the electrons by each impurity since the states are a superposition over many sites.

B. LDOS: $\rho_\alpha(\epsilon, i, \sigma)$

It is perhaps clearer if we look into greater detail into the LDOS, separating the spin components and trying to understand better the difference between the positive and the negative energy states.

Consider once again the case of a single impurity. Let us focus our attention on the first two levels, the first localized and the next in the continuum. Actually these constitute a set of 4 states due to the positive and negative energies. The analysis of the LDOS shows that for $J = 1$, considering first the positive energies, the first level has only a contribution from spin \uparrow and the first level with negative energy (symmetric to the other level) has only contribution from spin \downarrow . The magnitude of the spectral weight at the impurity site is different for the two states, as already noticed before²¹. In Fig. 18 we show the results for the boundstates for both $J = 1$ and $J = 2$. Considering now the second level located in the continuum, both at positive and negative energies, we obtain that there is a mixture of both spin components, even though in the case of the state at positive energy the magnitude of the peak is larger for the case of spin \uparrow than for the case of spin \downarrow and in the case of the state in the continuum at negative energy the relative magnitudes of the two spin components are reversed. Considering now the case of $J = 2$, where the level crossing has occurred, the nature of the states changes. The positive energy boundstate has now only a contribution from the spin \downarrow component and vice-versa, the first negative energy state has only contribution from the spin component \uparrow . As the level crossing occurred the spin content has changed. In Fig. 18 we only show the non-vanishing contributions. The other contributions vanish. On the other hand, in the first state in the continuum, where the two spin components contribute, the magnitude of the \uparrow component is now much larger than the \downarrow component while in the case of $J = 1$

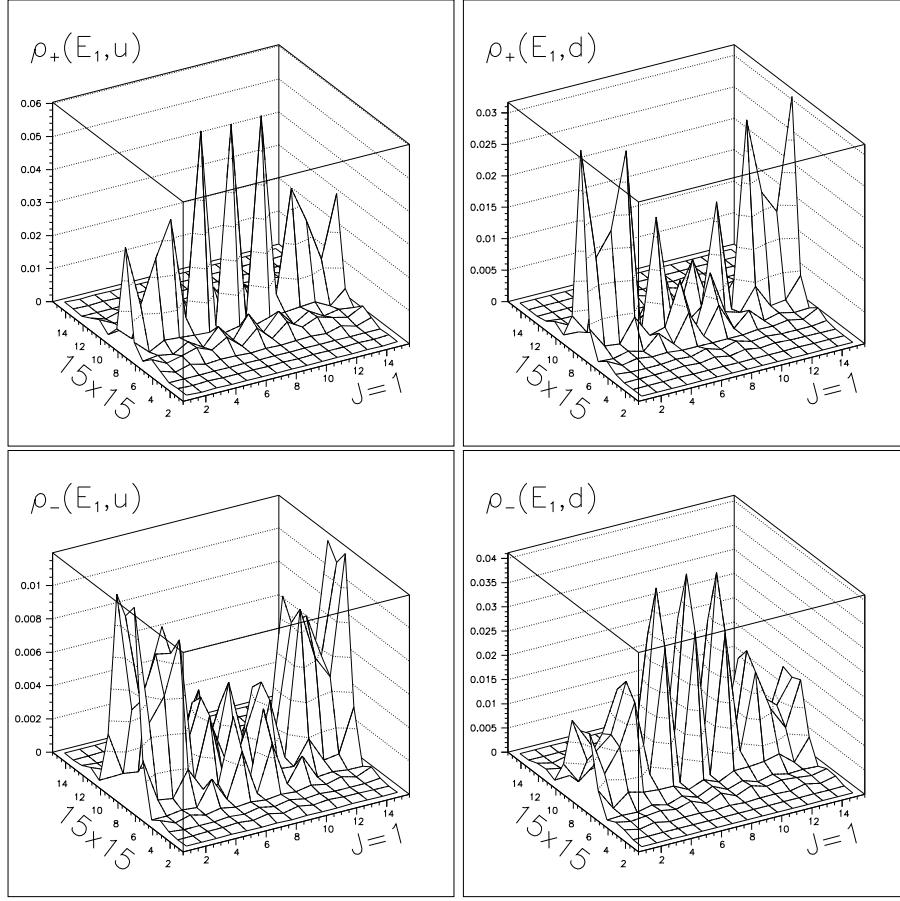


FIG. 19: LDOS for the domain wall DW1. a) $\rho_+(\epsilon_1, i, \uparrow)$ for $J = 1$, b) $\rho_+(\epsilon_1, i, \downarrow)$ for $J = 1$, c) $\rho_-(\epsilon_1, i, \uparrow)$ for $J = 1$, d) $\rho_-(\epsilon_1, i, \downarrow)$ for $J = 1$. (Note that in the vertical axis label $u = \uparrow$ and $d = \downarrow$).

the magnitudes were of similar size. Also, the \downarrow component of the second state of negative energy is now much smaller than the \uparrow component. The second state has to compensate for the spin flip of the lowest state by increasing the weight of the spin component aligned with the external impurity spin.

In the case of two impurities the behavior is similar to the case of one impurity, except that now the second state is also localized. Therefore, for $J = 1$ there are now two levels at positive energies with only spin \uparrow contribution and two states at negative energies with only spin \downarrow contribution. As one crosses to higher values of the coupling, for instance $J = 2$, the spin contributions for the boundstates change in a similar way to the single impurity case. For the set of parameters we consider here the two states are nearly degenerate and therefore the states change their nature basically simultaneously. Otherwise they would change their nature in succession²¹.

One may also consider a ferromagnetic chain where all the spins point in the z direction. This case is very similar to the single impurity case, as expected. All the positive energy states have the same spin content at low values of the coupling. If the coupling is large enough a similar situation will occur where all the boundstates have reversed their spin content.

The case of the DW1 is however different. Even at a small value of the coupling, in the sense that the first level crossing has not yet occurred, such as $J = 1$, the various states have a mixture of the two spin components. This is shown in Fig. 19. The boundstates at positive energy have a mixture of the two components, even though the magnitude at the impurity site is larger for the \uparrow component. For the negative energies the \downarrow component has a larger magnitude. In Fig. 19 we only show results for the lowest level but a similar trend is found for the other localized states. Note that both components have symmetric wavefunctions. This does not happen for instance for the case of the third level shown in Fig. 20. Even though the relative magnitudes of the spin components are the same, note that while the \uparrow component has a symmetric wave function the spin \downarrow has an anti-symmetrical wave function. For the third level at negative energy this is reversed.

Increasing the coupling to $J = 2$ does not change the nature of the first two level states. However, increasing further to $J = 4$ we see in Fig. 21 that there is once again a reversal of the magnitudes of the two spin components of the

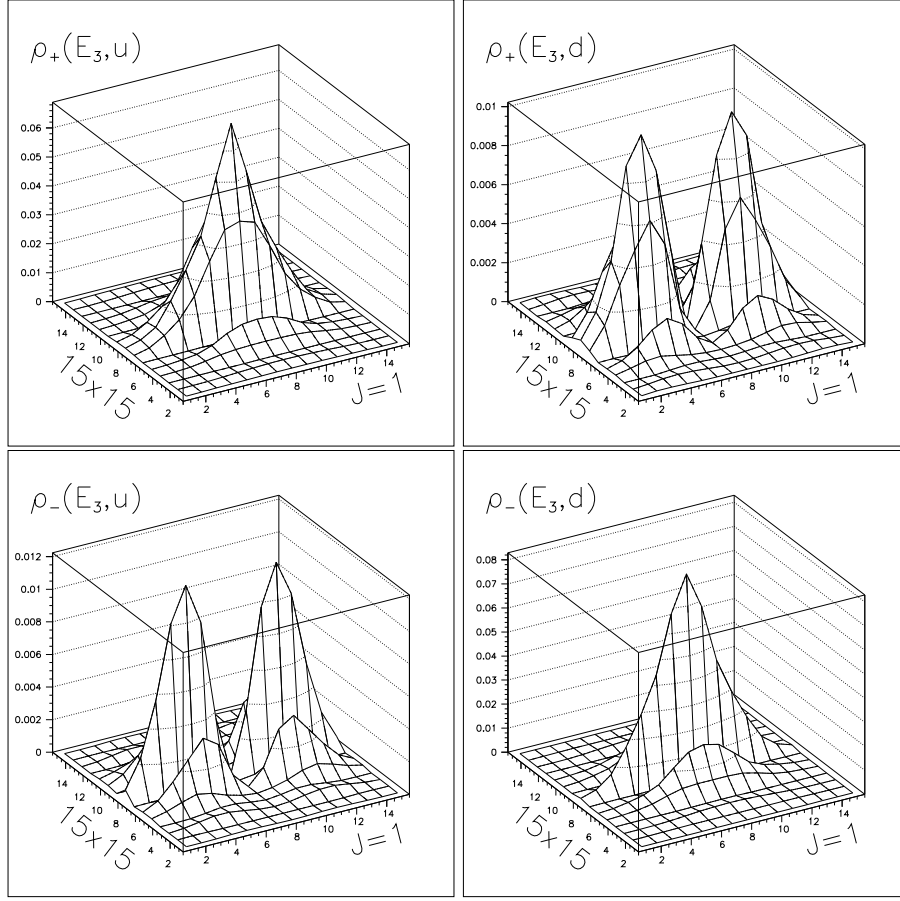


FIG. 20: LDOS for the domain wall DW1. a) $\rho_+(\epsilon_3, i, \uparrow)$ for $J = 1$, b) $\rho_+(\epsilon_3, i, \downarrow)$ for $J = 1$, c) $\rho_-(\epsilon_3, i, \uparrow)$ for $J = 1$, d) $\rho_-(\epsilon_3, i, \downarrow)$ for $J = 1$. (Note that in the vertical axis label $u = \uparrow$ and $d = \downarrow$).

first level. Even though both spin components contribute the magnitude of the \downarrow component is now larger than the magnitude of the \uparrow component. This also occurs for the second level. For the third level however, the two components have very similar magnitudes.

As we have seen in some cases the discrete states in the gap correspond to well defined spin polarized states. Then the QPT changes the spin polarization of these states leading to a sort of magnetic phase transition. However, in the case of DW1 domain wall, each discrete level corresponds to a mixture of spin up and down components because the spins in DW are oriented along axis x . In this case the QPT does not show explicitly the transition between spin up and down states. Nevertheless, the different spin components of the composition are also interchanged during the QPT. Thus, the analysis demonstrates that the QPT can be seen in the variation of magnetic state corresponding to the discrete levels. As for the origin of the QPT, we show that it is related only to the level crossing because at each crossing point the ground state is reconstructed.

C. Kinetic energy

To further confirm the nature of the states we have calculated at $T = 0$

$$\langle c_{i\sigma}^\dagger c_{j,\sigma'} \rangle = \sum_n \sigma \sigma' v_n(i, \sigma) v_n(j, \sigma') \quad (18)$$

We have considered two typical points in the system. The central point and a point far from the central line (in the bulk). Also we have considered both vertical (v) (along z) and horizontal (h) (along x) displacements of the electrons. In Figs. 22,23 we plot the hoppings for the cases of $\sigma, \sigma' = \uparrow$ and $\sigma, \sigma' = \downarrow$ as a function of the coupling. We have considered the cases of a single impurity, two impurities (four lattice sites apart), the domain wall DW1 and a ferromagnetic chain. In the bulk we expect the states to be extended independently of the coupling. This is so and

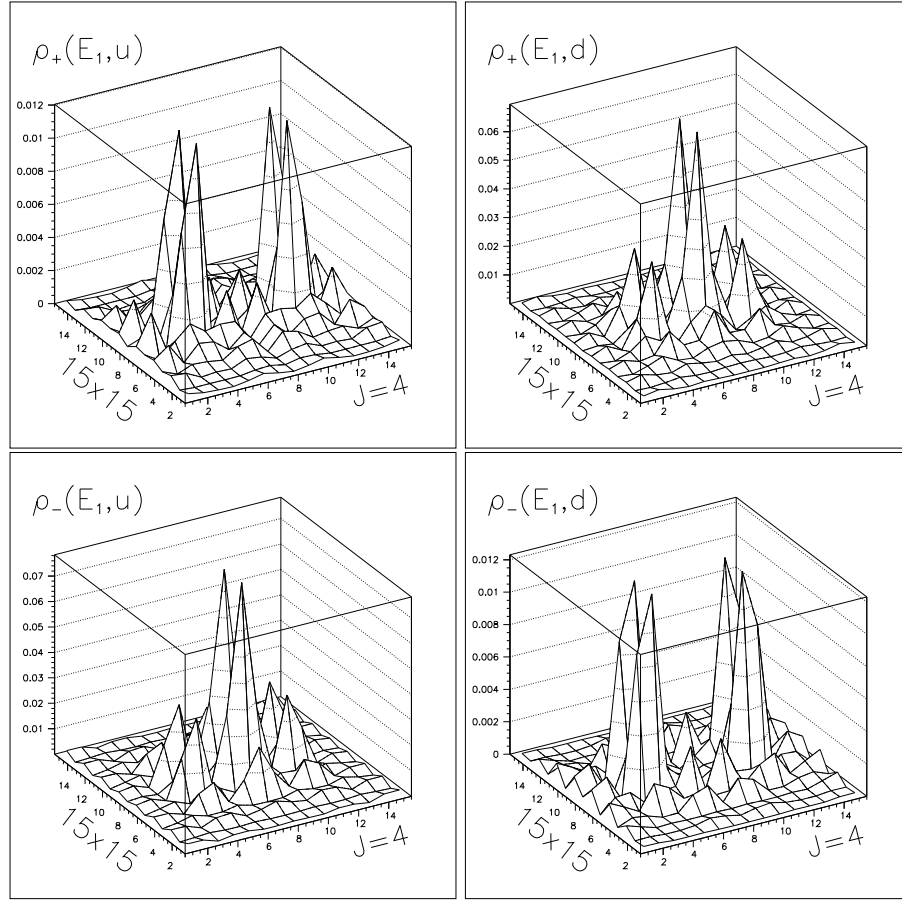


FIG. 21: LDOS for the domain wall DW1. a) $\rho_+(\epsilon_1, i, \uparrow)$ for $J = 4$, b) $\rho_+(\epsilon_1, i, \downarrow)$ for $J = 4$, c) $\rho_-(\epsilon_1, i, \uparrow)$ for $J = 4$, d) $\rho_-(\epsilon_1, i, \downarrow)$ for $J = 4$. (Note that in the vertical axis label $u = \uparrow$ and $d = \downarrow$).

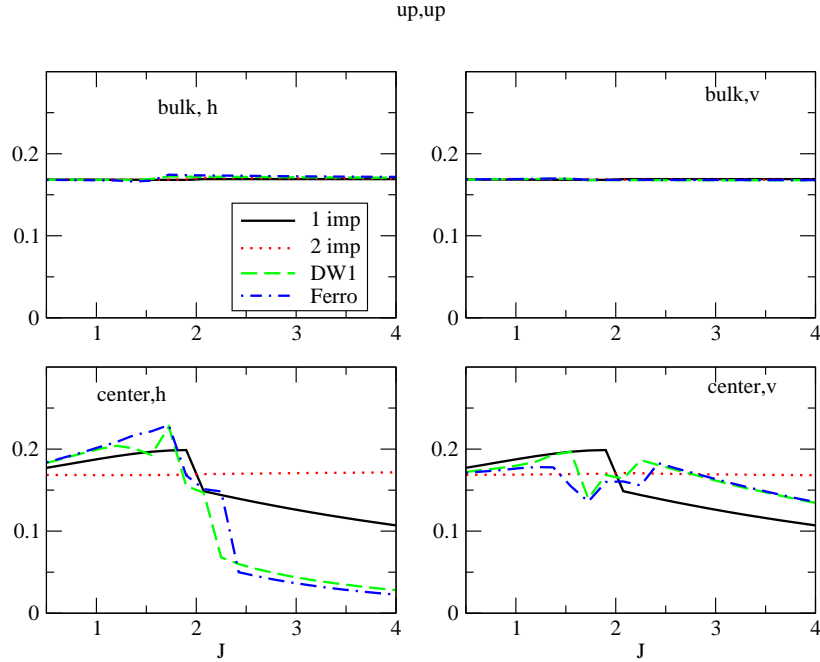


FIG. 22: Hopping matrix elements for up spin electrons from a bulk point and the center point, in the horizontal (along x) and vertical (along z) directions.

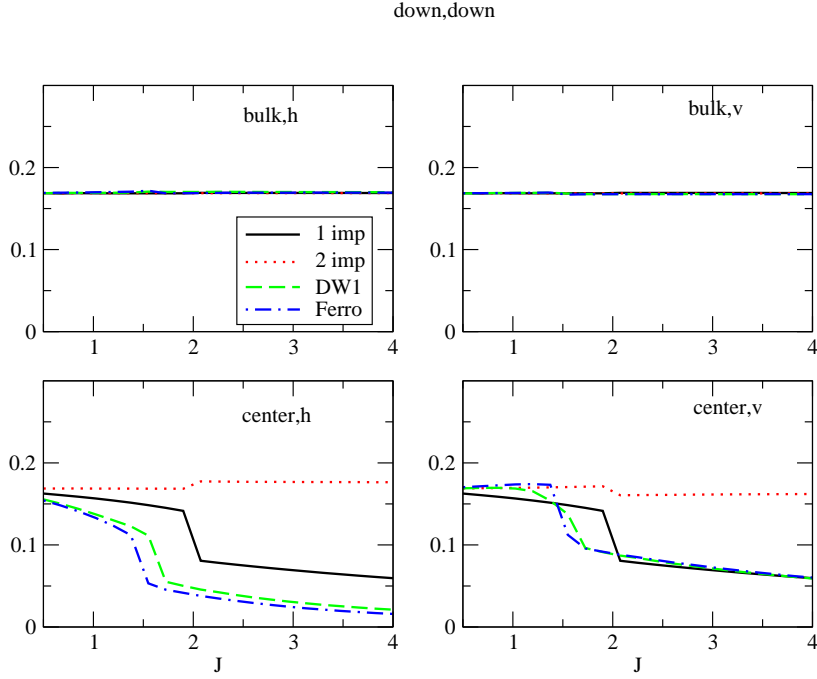


FIG. 23: Hopping matrix elements for down spin electrons from a bulk point and the center point, in the horizontal (along x) and vertical (along z) directions.

the hoppings are basically independent of J both along the vertical and horizontal directions and spin directions. The central point is different however and when various impurity spins are considered the horizontal and vertical directions are expected to be different. As shown in the Figs. 22,23 we see that as the coupling grows in general the hopping decreases, consistently with the “localized” nature of the states. This is particularly so along the chain direction, for the lines of spins. In the case of two impurities, and due to the rather local nature of the effect of the impurity spins, the hoppings at the central point do not change much as a function of the coupling. As the \uparrow spins are in majority the hopping term for the cases of \uparrow, \uparrow are typically larger. Interestingly, and even though the hopping term of the Hamiltonian is diagonal in the spin, in the case of the domain wall $DW1$ there is a non-vanishing spin flip term, even though quite small.

V. STABILITY OF DOMAIN WALL

So far we have assumed that the domain wall is stable. We may solve this stability issue self-consistently and study the stability of the domain wall. This may be achieved introducing effective interactions between the impurity spins, possibly mediated by the quasiparticles. Let us approximate these interactions by an Heisenberg like term. The part of the Hamiltonian involving the classical spins may be written in mean field as

$$- \sum_i J (S_i^x < \sigma^x >_i + S_i^z < \sigma^z >_i) - \frac{1}{2} \sum_{<i,j>} J_f (S_i^x S_j^x + S_i^z S_j^z) \quad (19)$$

where we consider only the coupling between nearest neighbors, assumed to be ferromagnetic for simplicity. In the mean field approximation for both electrons and spins, the external impurity spins are determined from their mutual interactions and the interaction with the average spin density of the electrons. We look for the equilibrium impurity spin configuration minimizing the energy with respect to the impurity spin components. This equilibrium distribution is then inserted in the Bogoliubov-de Gennes equations and solved self-consistently. The impurity spins satisfy the constraint that $(S_i^x)^2 + (S_i^z)^2 = (\vec{S}_i)^2$. An equivalent way is to minimize the Hamiltonian with respect to the angles themselves, since in this way the constraint is automatically enforced. Minimizing with respect to the angles φ_l we get the set of equations

$$\begin{aligned} & - J \langle \sigma^z \rangle_i \cos \varphi_i + J \langle \sigma^x \rangle_i \sin \varphi_i \\ & + J_f \sin \varphi_l \cos \varphi_{i+\delta} - J_f \cos \varphi_i \sin \varphi_{i+\delta} = 0 \end{aligned} \quad (20)$$

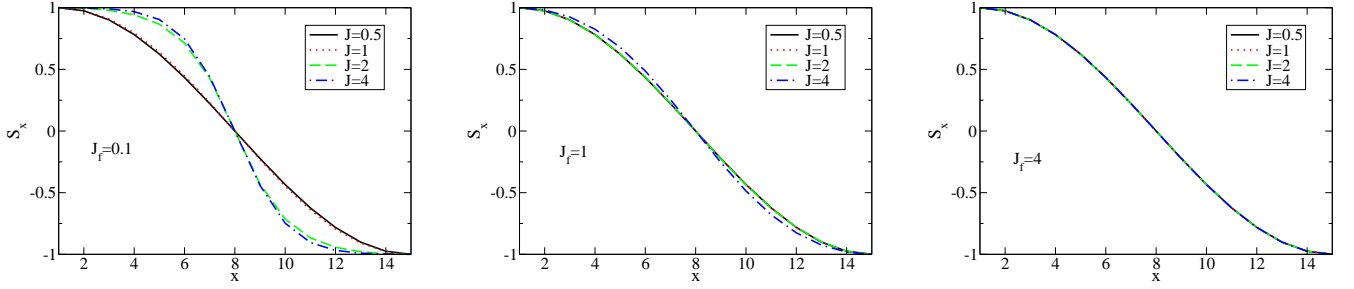


FIG. 24: Profile of S_x of the impurity spins for DW1 and for $J_f = 0.1, 1, 4$ as a function of J .

where $\delta = \pm 1$. These equations hold at each impurity site. The solution leads to a stable domain wall whose shape depends on $\frac{J}{J_f}$. The results are shown in Fig. 24. We see that the profiles are stable. In the case of a small coupling between the impurity spins ($J_f = 0.1$), for small J the profile is tending to a large λ , defined in Eqs. (14,15), which implies an almost linear profile between the end spins. As J increases the value of λ decreases and there seems to be a rapid variation between $J = 1$ and $J = 2$. Increasing the value of J_f , the stability is improved. For large J_f the profile is almost independent of J . Also note that increasing J_f the value of λ increases and the profile becomes almost linear for any value of J .

We should note that one strictly does not have to introduce an effective Heisenberg interaction between the impurity spins, if we were to treat them as quantum mechanical. The Kondo interaction would couple the impurity spins to the conduction electron spin density, which in turn would give rise dynamically to an effective long-range interaction between the impurity spins. In our mean-field approach this interaction is introduced phenomenologically, much in the same way as the attractive interaction between the electrons to give rise to pairing. While a mean-field approach (BCS) is quite good for the superconductivity order, the correct description of the Kondo effect is much more involved as well as a proper treatment of the coupling between the impurity spins. However, as shown before in a similar context⁶⁹ the results are quite similar.

VI. EFFECT OF TEMPERATURE

In this section we briefly study the effect of temperature on the results. Clearly the quantum phase transitions are smeared out but the same trends prevail.

In Fig. 25 we show the order parameter and the spin density for a single impurity and the DW1 as a function of temperature for the two typical cases of $J = 1, 2$. We see clearly that the critical temperature is basically the same when we increase the number of spins. Note the critical temperature for one impurity and the DW1 at about $T \sim 0.5 - 0.6$. However, the behavior of the spin density is quite different as a result of the underlying phase transition. In the case of $J = 2$ the spin density is finite and then decreases while for $J = 1$ the increase in temperature increases the density due to excitation to higher levels. Note for both cases the different starting point of s_z^T due to the quantum phase transition when we go from $J = 1$ to $J = 2$.

Clearly, as shown in Fig. 26, the quantum transitions between the plateaus of the spin densities as a function of the coupling are now smeared, but the same overall trend persists. Even though the temperature smoothens the curves and the quantum phase transitions disappear (they only occur at $T = 0$) the behaviors are robust to temperature. In particular the regimes where at $T = 0$ there is a finite value for one of the average magnetizations it persists at small finite temperatures.

VII. SUMMARY

In this work we considered the effect of correlated magnetic impurities inserted in a conventional superconductor. Previous results on few impurities revealed the existence of sequences of quantum phase transitions, associated with level crossings, that lead to discontinuous changes in the properties of the system, such as the magnetization. We extended these results to the case when we have sets of impurity spins that are correlated through mutual interactions, that may be originated via RKKY-like interactions mediated by the electrons. In particular, we considered cases where the impurity spins are organized in such a way that form domain walls which, to simplify, we have limited in this work to one-dimensional arrays of spins inserted in the superconductor. These domain wall structures may be obtained imposing different boundary conditions at different sides of the mesoscopic systems here considered. As in the case of

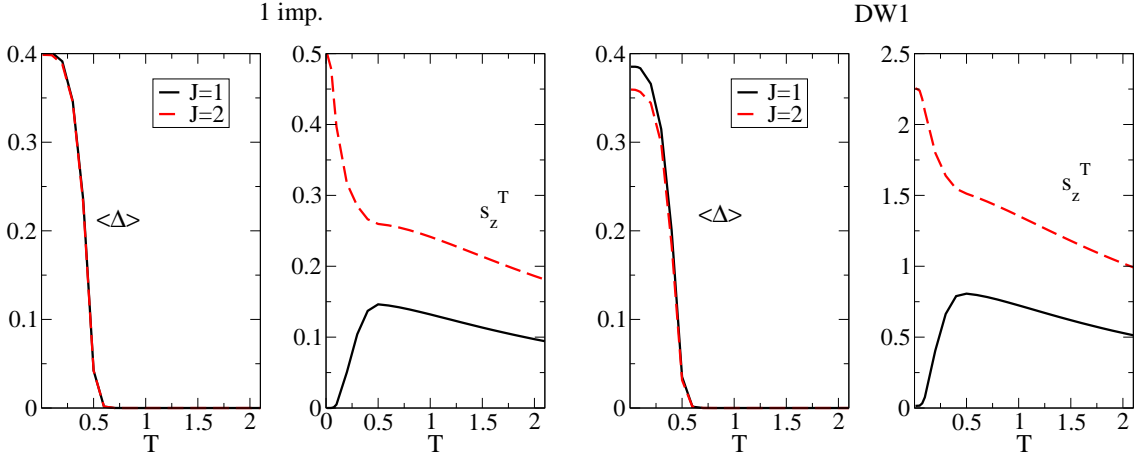


FIG. 25: $\langle \Delta \rangle$ and s_z^T for one impurity and DW1 as a function of temperature for various values of J .

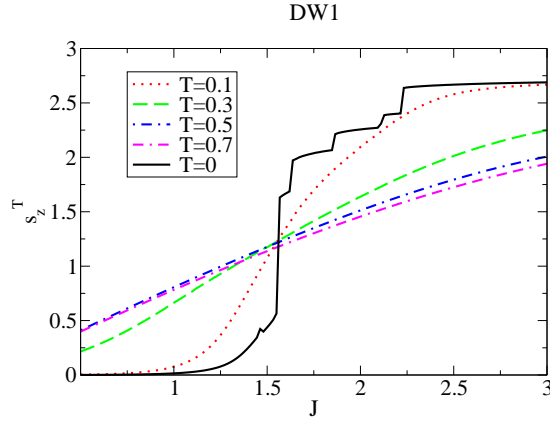


FIG. 26: Effect of temperature on the results for the various magnetizations (total and left or right) for DW1.

a few impurities, we found a series of quantum phase transitions that we have analyzed. In general the introduction of foreign objects in the superconductor originates interference effects. These are revealed in the LDOS. We have presented detailed results which show the complex nature of the multiple interference effects.

We have also shown that the domain wall structures considered here are stable taking into account the mediated interactions between the impurity spins. To simplify we considered in this work classical spins. The case of a full quantum problem where the impurity spins are described by quantum operators leads to Kondo like effects in the superconductor. This is much more involved and the use of the Bogoliubov-de Gennes formalism requires that we take this classical limit. Fortunately, at least if the coupling between the electronic spin density and the impurity spins is not large, the classical description is enough.

In this work we focused on the effect of the impurity spins on the superconductor. The opposite problem of the effect of the superconductor on the impurity spins may be interesting if, for instance by passing a current through the superconductor, the spin torque created by a spin polarized current on the impurity spins changes their relative orientations. This is a problem that has received much attention in the context of spintronics, where spin polarized currents passing through a magnetic semiconductor move the position of the domain walls. The related problem in the context of the magnetic correlated impurities spins in the superconductor will be considered elsewhere.

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